NOTIZEN

The Hall Side Resistance of a Hall Generator

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Endsley et al. [1] have derived an expression for the Hall side resistance Z_{22} of a Hall generator, assuming that the width of the contact strips is negligibly small compared to the length of the sample. For certain applications, however, this assumption may not hold good and, therefore, in such cases the expression derived by these authors is no longer valid. In the present note, an attempt is made to derive an expression for Z_{22}, free from the above assumption. Incidentally, such an expression would also be valid for calculating the resistance between two small area rectangular contacts placed on the opposite edges of a thin rectangular semiconductor slab.

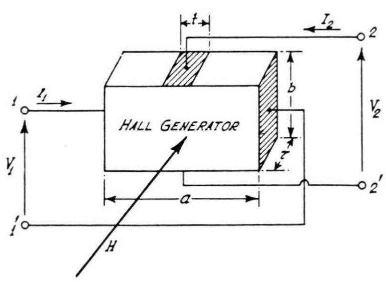

Fig. 1. A schematic diagram of the Hall Generator.

Following Wick [2], the Hall generator (shown in Fig. 1) can be transformed into the structure, shown in Fig. 2, with the help of the equation,

$$\zeta = \int_0^z \frac{dz}{[(z^2-\varkappa_b^2)(z^2-1/\varkappa_b^2)]^{1/2}} . \tag{1}$$

Integrating eqn. (1) within proper limits, the dimensions of the Hall generator may be shown to be related

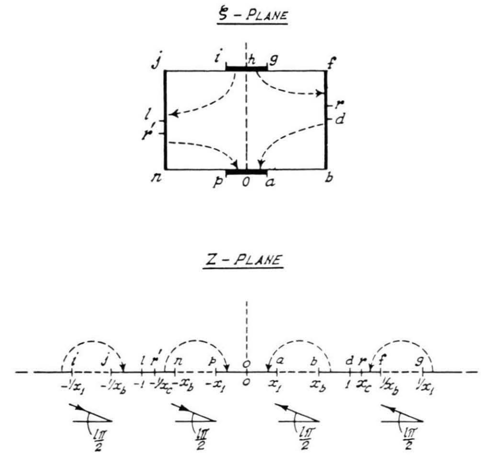

Fig. 2. Diagram showing the transformation of the Hall Generator in ζ-plane to the real axis of the z-plane.

to the z-plane parameters as follows:

$$a/b = 2\,K(k)/K(k'), \tag{2}$$

and

$$t/b = 2\,F(k,\varphi)/K(k'), \tag{3}$$

where, $k=\varkappa_b^2$, $k'=\sqrt{1-k^2}$, $\sin\varphi = \varkappa_1/\varkappa_b$ and K and F stand respectively for complete and incomplete Elliptic integrals of the first kind.

Now, according to Wick, the Hall side impedance Z_{22} and the transfer impedance Z_{21} are respectively given by

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{M-N}{\sigma\,\tau\,P\cos l\,\pi/2}, \tag{4}$$

and

$$Z_{21} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{M+N}{\sigma\,\tau\,P\cos l\,\pi/2}, \tag{5}$$

where,

$$M \equiv \int_{\varkappa_1}^{\varkappa_b} (\ )\,d\varkappa, \quad N \equiv \int_{1/\varkappa_b}^{1/\varkappa_1} (\ )\,d\varkappa, \quad \text{and} \quad P \equiv \int_{-\varkappa_1}^{\varkappa_1} (\ )\,d\varkappa,$$

the integrand in each case being

$$\frac{(z^2-1-\alpha\,z)\,dz}{[(z-\varkappa_1)(z+1/\varkappa_1)(z+\varkappa_b)(z-1/\varkappa_b)]^{(1+l)/2}[(z+\varkappa_1)(z-1/\varkappa_1)(z-\varkappa_b)(z+1/\varkappa_b)]^{(1-l)/2}} . \tag{6}$$

[1] D. L. Endsley et al., Trans. IRE on Electron Devices, May 1961, p. 220.

[2] R. F. Wick, J. Appl. Phys. **25**, 741 [1954].

In eqn. (6)

$$l = 2/\pi\,\Theta\,,$$

and, $$a \equiv \varkappa_c - 1/\varkappa_c = \int\limits_{\varkappa_b}^{1/\varkappa_b} \frac{\varkappa^2-1}{|D|}\,d\varkappa \Big/ \int\limits_{\varkappa_b}^{1/\varkappa_b} \frac{\varkappa\, d\varkappa}{|D|}, \qquad (7)$$

where Θ stands for HALL angle and $|D|$ represents the magnitude of the denominator of eqn. (6).

The integrations indicated above are in most cases quite difficult, and can only be done by the laborious numerical method. In the present case, however, as we are not interested in the effect of magnetic field, the expressions given above can be simplified considerably. Since there is no magnetic field, both the HALL angle Θ (i.e., l) and the transfer impedance Z_{21} are zero. Utilizing these relations, eqn. (4) reduces to

$$Z_{22} = 2\,M/(\sigma\,\tau\,P)$$

or $$Z_{22}\,\sigma\,\tau = 2\,M/P, \qquad (8)$$

where, now, $M = \int\limits_{\varkappa_1}^{\varkappa_b} \frac{\varkappa^2-1}{|D|}\,d\varkappa$ and $P = \int\limits_{-\varkappa_1}^{\varkappa_1} \frac{\varkappa^2-1}{|D|}\,d\varkappa$.

$Z_{22}\,\sigma\,\tau$ has been chosen instead of Z_{22}, as the former is a convenient dimensionless quantity. Integrating the expressions for M and P, and substituting in eqn. (8), one obtains

$$Z_{22}\,\sigma\,\tau = \frac{2\,M}{P} = \frac{K(\cos^{-1}\beta_1/\beta_2)}{K(\sin^{-1}\beta_1/\beta_2)}, \qquad (9)$$

where, $\beta_1 = \varkappa_b + 1/\varkappa_b$ and $\beta_2 = \varkappa_1 + 1/\varkappa_1$. From eqns. (9), (4) and (5) the dependence of $Z_{22}\,\sigma\,\tau$ on the dimensions of the sample and the HALL strips can be studied.

In Fig. 3 $Z_{22}\,\sigma\,\tau$ has been plotted against a/b, with t/b as parameter. It is observed that $Z_{22}\,\sigma\,\tau$ tends to attain a saturating value, for values of $a/b \geqq 2.0$.

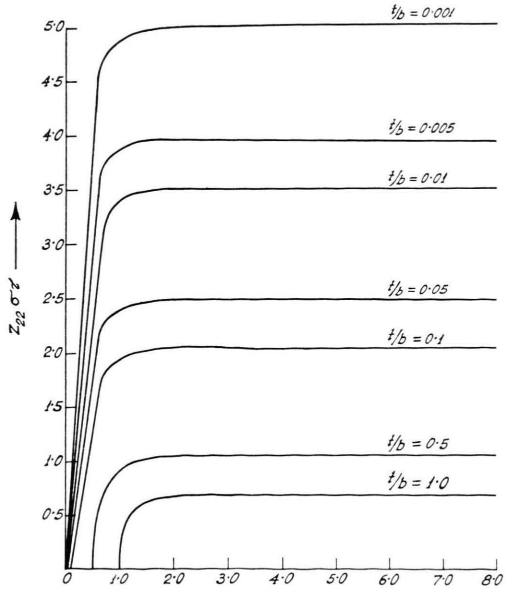

Fig. 3. Variation of $Z_{22}\,\sigma\,\tau$ with a/b for different values of t/b.

Below this range, $Z_{22}\,\sigma\,\tau$ decreases rapidly with a/b, and ultimately becomes zero at $a/b = t/b$. The saturating values of $Z_{22}\,\sigma\,\tau$ for different values of t/b, agree with those calculated from ENDSLEY'S expression. This shows that for $a/b \geqq 2.0$, the sample behaves essentially as an infinite slab of the type assumed by ENDSLEY et al.

Fig. 4 shows values of $Z_{22}\,\sigma\,\tau$ plotted against t/b with a/b as parameter. As is to be expected from the above discussion, the curves are found to posses a tendency to coalesce for values of $a/b \geqq 2.0$. It is also observed that for t/b approaching zero, all the curves tend to infinity, irrespective of the value of a/b. As t/b increases, $Z_{22}\,\sigma\,\tau$ falls exponentially at the outset, but as t/b becomes comparable with a/b, the occurrence of a point of in-

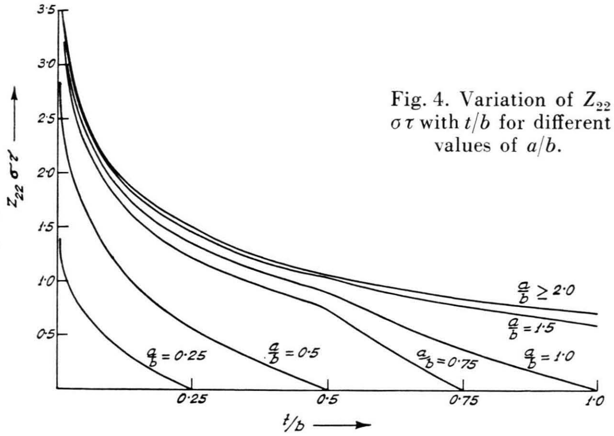

Fig. 4. Variation of $Z_{22}\,\sigma\,\tau$ with t/b for different values of a/b.

flexion can be noticed for values of $a/b \approx 1$. For $a/b \geqq 2$ it is further seen that there is a certain range of values of t/b, over which the variation of $Z_{22}\,\sigma\,\tau$ is quite small. The designer of a HALL generator would obviously choose t/b within such a range in order to minimise production spread of the values of output impedance of the device although the output impedance itself would be rather small. For many applications, however, a larger output impedance may be desirable which requires a smaller value of the width of the contact strip and consequently causes a sharper variation of $Z_{22}\,\sigma\,\tau$ with t/b. Thus a compromise has to be made between the above two considerations in choosing the proper width of the HALL strips, so that the HALL generator may have a high output impedance and at the same time a low production spread of this parameter.

In deriving eqn. (8) the HALL strips have been assumed to be perfectly aligned. In practice, however, this is very difficult to realise. As a result, there is always a so-called "off-set" voltage between the HALL strips even in the absence of any magnetic field. This means that Z_{21} in this case will have a finite value and therefore $M \neq -N$. It can be shown that the effect of this lack of alignment is to increase the effective value of t/b and thus, to reduce the value of $Z_{22}\,\sigma\,\tau$. Further studies along these lines are in progress and will be reported in due course.

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